Solving PDEs with Radial Basis Functions

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Radial basis functions provide flexible and powerful tools for the reconstruction of unknown functions from scattered samples or measurement. Their meshfree nature allows the application to the solution of a variety of problems on irregular domains, possibly in high-dimensional spaces or on manifolds.

The approximation space is usually formed using the shifts of a fixed basis function. This simple approach makes it easy to construct approximation spaces of arbitrary smoothness and in arbitrary dimensions. It is also possible to incorporate physical features like incompressibility into the approximation space.

In this talk, I will introduce various ways of solving PDEs numerically using RBFs. I will address collocation and Galerkin techniques for elliptic and parabolic problems. I will discuss error and stability estimates and give several examples. I will particularly discuss the solution of PDEs on (possibly evolving) manifolds and the solution of the Navier-Stokes equations.